



Hausdorff School "Recent developments in disordered systems"

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organized by Johannes Alt, Luca Fresta, Michel Pain

Abstracts

David Belius (UniDistance Suisse)

A brief introduction to spin glasses and the TAP

Abstract: The goal of this talk is to:

- explain what spin glass models are and why they are interesting,
- give an overview of some known results and open questions,
- give an introduction to the TAP approach to studying spin glass models.

Giorgio Cipolloni (University of Arizona)

Logarithmically correlated fields in non-Hermitian random matrices.

Abstract: We study the Brownian evolution of large non-Hermitian matrices and show that their log-determinant converges to a 2+1 dimensional Gaussian field in the Edwards-Wilkinson regularity class, i.e. logarithmically correlated for the parabolic distance. This gives a dynamical extension of the celebrated result by Rider and Virag (2012) proving that the fluctuations of the eigenvalues of Gaussian non-Hermitian matrices converge to a 2 dimensional log-correlated field. Our result, previously not known even in the Gaussian case, holds out of equilibrium for general matrices with i.i.d. entries. We also study the extremal values of these fields and demonstrate their logarithmic dependence on the matrix dimension.

Alice Guionnet (Ecole Normale Supérieure Lyon)

Large deviations for the largest eigenvalue of large random matrices, and applications.

Abstract: In this mini-course I will review several recent large deviation results for the largest eigenvalue of large random matrices, and their relations with spin glasses and the study of the volume of local minima of random functions. This is based on joint works with G. Ben Arous, J. Boursier, N. Cook, A. Dembo, R. Ducatez, J. Husson, and M. Maida.

Ioan Manolescu (University of Fribourg)

An invitation to FK percolation.

Abstract: In the last decades, FK percolation established itself as a central model within statistical mechanics, both for its relation to other models as well as for its intrinsic behavior. This mini-course will mostly be dedicated to FK-percolation on the square lattice \mathbb{Z}^2 and will specifically touch on the sharpness of the phase transition, its continuity/discontinuity and properties of the critical phase. If time permits, we will explore the connection between FK-percolation and the six-vertex model and sketch an approach to proving a GFF scaling limit for the height-function associated to the latter.

Vieri Mastropietro (University of Rome La Sapienza)

Interacting fermionic systems with quasi-periodic disorder.

Abstract: Disorder can be represented as a random variable or by a quasi-periodic potential, and the second description is realized in quasi-crystals or cold atoms experiments. The interplay of quasi-periodic disorder and many-body interaction in fermionic systems at zero temperature can be investigated by Renormalization Group methods. Small divisors similar to the ones in Kolmogorov-Arnold-Moser (KAM) theory appear, but the presence of interaction produces complex phenomena related to the presence of loops in the perturbative expansion (in KAM only tree graphs appear). I will review some rigorous results for a number of systems including 1d interacting fermionic systems with strong or weak disorder, Weyl semimetals and Ising model with quasi-periodic hopping.

- [1] V. Mastropietro Localization of interacting fermions in the Aubry-Andre model Phys. Rev. Lett. 115, 180401 (2015)
- [2] V. Mastropietro: Localization in the ground state of an interacting quasi-periodic fermionic chain Comm. Math. Phys. 342, 1, 217-250 (2016) 19
- [3] V. Mastropietro Localization in Interacting Fermionic Chains with Quasi-Random Disorder Comm. Math. Phys. 351, 283309(2017)
- [4] V. Mastropietro Small Denominators and Anomalous Behaviour in the Incommensurate HubbardHolstein Model Communications in Mathematical Physics volume 201, pages81115 (1999)
- [5] V. Mastropietro Persistence of gaps in the interacting anisotropic Hofstadter model Phys. Rev. B 99, 155154 2019
- [6] V. Mastropietro Stability of Weyl semimetals with quasiperiodic disorder Phys. Rev. B 102, 04510 2020
- [7] M Gallone V. Mastropietro Universality in 2d quasiperiodic Ising model and Harris-Luck irrelevance Comm Math Phys (in print)

Eliran Subag (Weizmann Institute of Science, Israel)

From optimization of spin glasses to solving random polynomial systems.

Abstract: In these talks I will discuss two algorithmic problems and some connections between them. I will start with the problem of optimizing a mixed p-spin Hamiltonian, where I will focus on the spherical case – i.e., the problem of designing an efficient algorithm for minimizing a random polynomial function over the sphere in high dimensions. It turns out that the same random polynomials

have been considered by Shub and Smale in the 90s in the context of algorithms for solving random polynomial systems. There too, in the homogeneous case, one is interested in finding solutions on the sphere. I will discuss recent joint works with Andrea Montanari on Smale's 17th problem over the reals and related algorithmic questions.

Simone Warzel (Technical University of Munich)

An invitation to quantum spin glasses

Abstract: One of the most many-faceted areas in the statistical mechanics of disordered systems are spin glasses. Classical spin glass models, such as the one by Sherrington-Kirkpatrick (SK) were originally introduced to describe the unusual magnetic properties of some metal alloys with competing ferro- and anti-ferromagnetic interactions in which the magnetic spins of the component atoms are not aligned in regular pattern. Spin configurations in such models exhibit frustration and/or an emerging hierarchical organization. From a mathematical perspective, mean-field spin glasses such as the SK model have triggered a well-known subfield of probability theory. Much of the structure predicted by physicists could be transformed into mathematical technology. This stands in contrast to studies of the fate of spin glass physics concerning quantum effects induced by e.g. a transversal field. This has been a topic of continuing interest in the physics community since the 1990s. Mathematical studies have however picked up this only field fairly recently. In my lectures, I will give an introduction to the field, an overview of some of the results, as well as the techniques behind them.